

8.1 Introduction to Functions

Finding the Domain and Range of a Relation

Definition of a Relation

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the domain of the relation, and the set of all second components of the ordered pairs is called the range of the relation.

Example 1: Find the domain and range of the relation.

$$\{(2002, 364.2), (2003, 424.9), (2004, 559.7), (2005, 607.1)\}$$

$$\text{Domain: } \{2002, 2003, 2004, 2005\}$$

$$\text{Range: } \{364.2, 424.9, 559.7, 607.1\}$$

The ordered pairs above represent the median housing prices in San Diego metro area for the years 2002 through 2005. The first number in each ordered pair is the year and the second is the median house price in the San Diego metro area for that year.

Determining Whether a Relation is a Function

Definition of a Function:

A function is a relation in which each member of the domain corresponds to exactly one member of the range. No two ordered pairs of a function can have the same first component and different second components.

Example 2: Determining Whether a Relation is a Function

To determine whether a relation is a function, determine if any domain element corresponds to more than one range element.

- $\{(2,6), (3,7), (4,5), (1,9)\}$; Function; each domain element corresponds to exactly one range element.
- $\{(2,6), (3,7), (2,5), (1,9)\}$; Relation - Not a Function; "2" corresponds to both "6" and "5."
- $\{(2,6), (3,6), (4,5), (1,9)\}$; Function; each domain element corresponds to exactly one range element.

Functions as Equations and Function Notation

Functions are often given in terms of equations rather than as sets of ordered pairs. Consider the equation

$$y = \frac{250(3x+5)}{x+25}$$

where the variable x represents time elapsed in years and the variable y represents the elk population at the end of x years. The variable x is called the independent variable and the variable y is called the dependent variable because its value depends on x . Because the equation represents a function, we say that y is a function of x , and we write $y = f(x)$. The notation $f(x)$ represents the range value that corresponds to the domain value of x .

Example 3: Consider the equation $y = \frac{250(3x+5)}{x+25}$ given in the paragraph above.

a. Is y a function of x ? Explain. *Yes, each year should correspond to exactly one measured elk population.*

b. Write the equation in function notation.

$$f(x) = \frac{250(3x+5)}{x+25}, \text{ where } y = f(x)$$

c. Find $f(3)$ and interpret the meaning of your result.

$$f(3) = \frac{250 \cdot [3(3)+5]}{(3)+25}, \quad f(3) = \frac{3500}{28}$$

$$f(3) = \frac{250 \cdot (14)}{28}, \quad f(3) = 125$$

After 3 years, the measured elk population is 125.

Example 4: Find the indicated function value:

a. $f(2)$ if $f(x) = 3x + 7$

$$f(2) = 3(2) + 7,$$

$$f(2) = 6 + 7,$$

$$f(2) = 13$$

b. $f(-1)$ if $f(x) = 2x^2 - 7x$

$$f(-1) = 2(-1)^2 - 7(-1),$$

$$f(-1) = 2 \cdot 1 + 7,$$

$$f(-1) = 2 + 7,$$

$$\underline{f(-1) = 9}$$

c. $f(q)$ if $f(x) = \frac{x}{x^2 + 1}$

$$f(q) = \frac{(q)}{(q^2 + 1)}$$

$$f(q) = \frac{q}{q^2 + 1}$$

d. $f(a+h)$ if $f(x) = x^2 - 3x + 2$

$$f(a+h) = (a+h)^2 - 3(a+h) + 2,$$

$$f(a+h) = [a^2 + 2ah + h^2] - 3a - 3h + 2,$$

$$f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 2$$

S.D.W.K

$$(a+h)^2 = (a+h)(a+h)$$

$$= a^2 + ah + ah + h^2$$

$$= a^2 + 2ah + h^2$$

8.2 Graphs of Functions and the Vertical Line Test

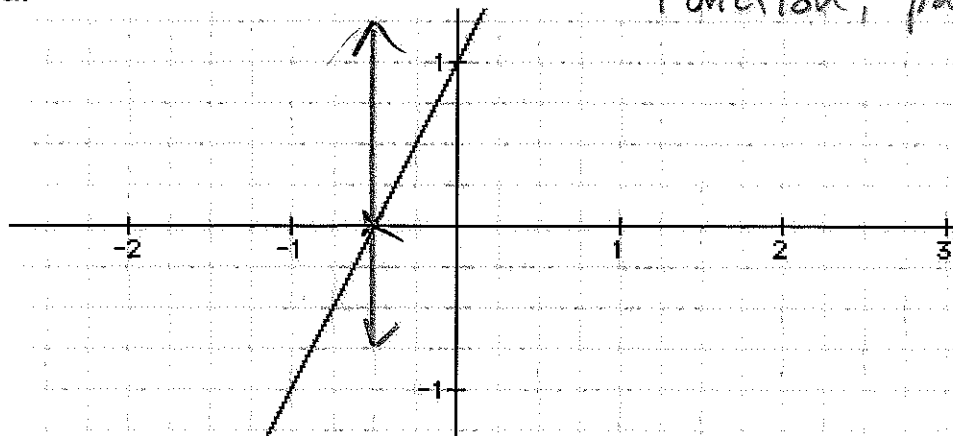
The graph of a function or an equation is the graph of its ordered pairs. Not every graph represents a function. If a graph contains two or more different points with the same first coordinate, the graph cannot represent a function. Points that share a common first coordinate are vertically above or below each other. This observation is the basis of a test that is useful in determining if a given graph defines y as a function of x (page 523).

Vertical Line Test for Functions

If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x .

Example 5: Use the vertical line test to identify graphs in which y is a function of x .

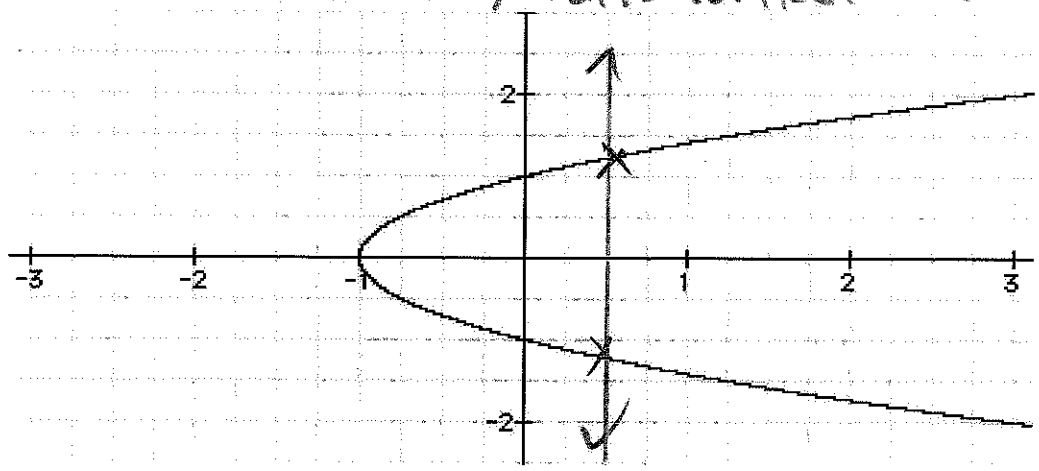
a.



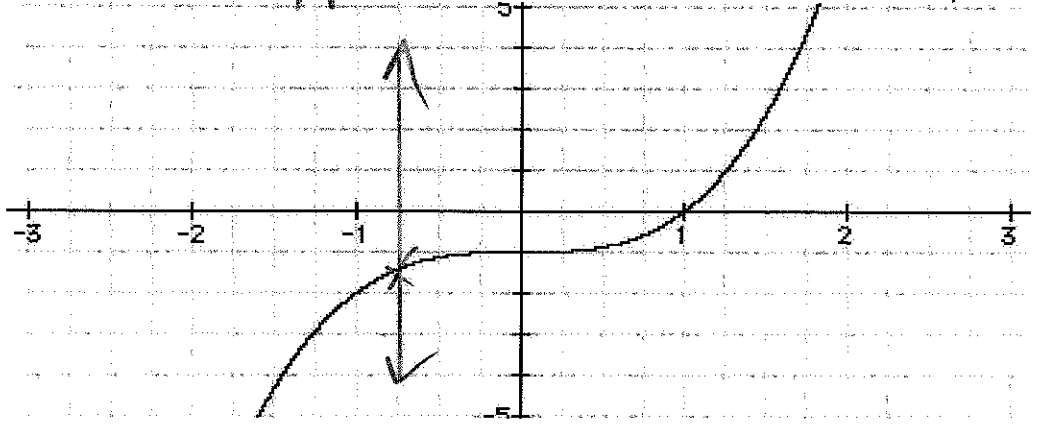
Function, passes vertical line test.

Ex 5:

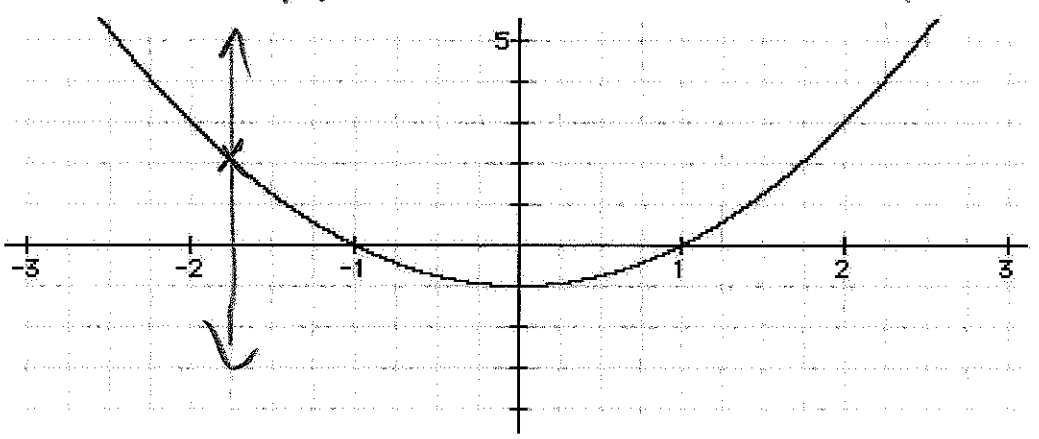
b. Not a function, fails vertical line test.



c. Function, passes vertical line test.



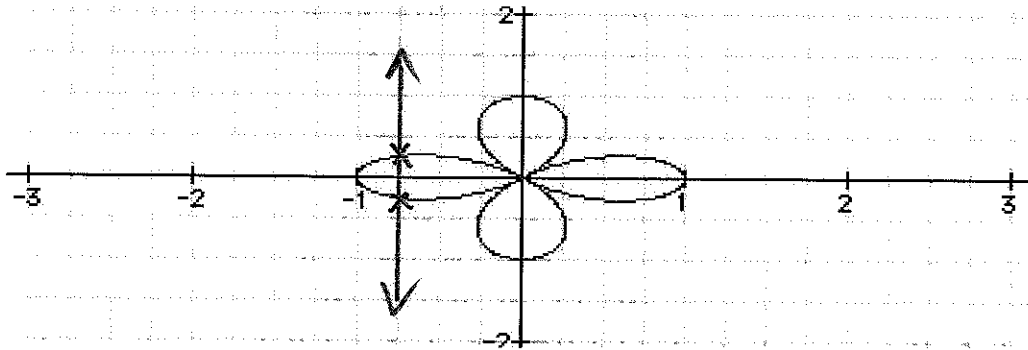
d. Function, passes vertical line test.



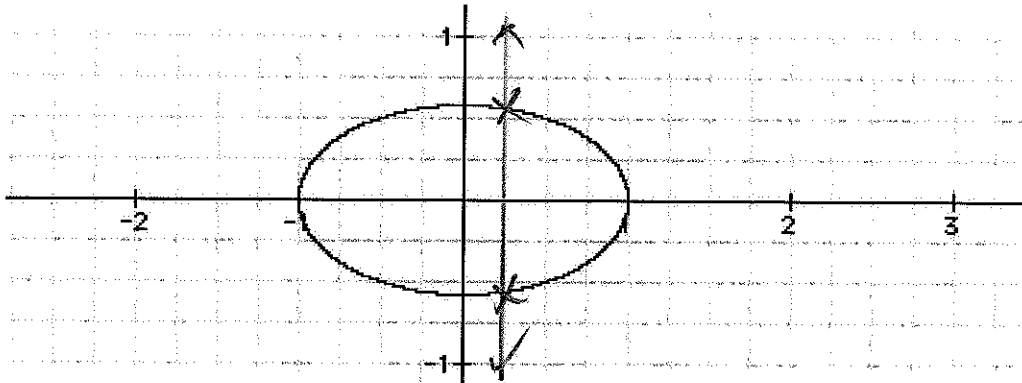
Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

EES!

e. Not a function, fails vertical line test.



f. Not a function, fails vertical line test.



Obtaining Information from Graphs

You can obtain information about a function from its graph. At the right or left of a graph, you will often find closed dots, open dots, or arrows.

1. A closed dot indicates that the graph does not extend beyond this point and the point belongs to the graph.
2. An open dot indicates that the graph does not extend beyond this point and the point does not belong to the graph.
3. An arrow indicates that the graph extends indefinitely in the direction in which the arrow points.

Note!

$$f(20) = 0.4(20)^2 - 36(20) + 1000$$

$$f(20) = 0.4(400) - 720 + 1,000$$

$$f(20) = 160 - 720 + 1,000$$

$$f(20) = -560 + 1,000$$

$$f(20) = 440$$

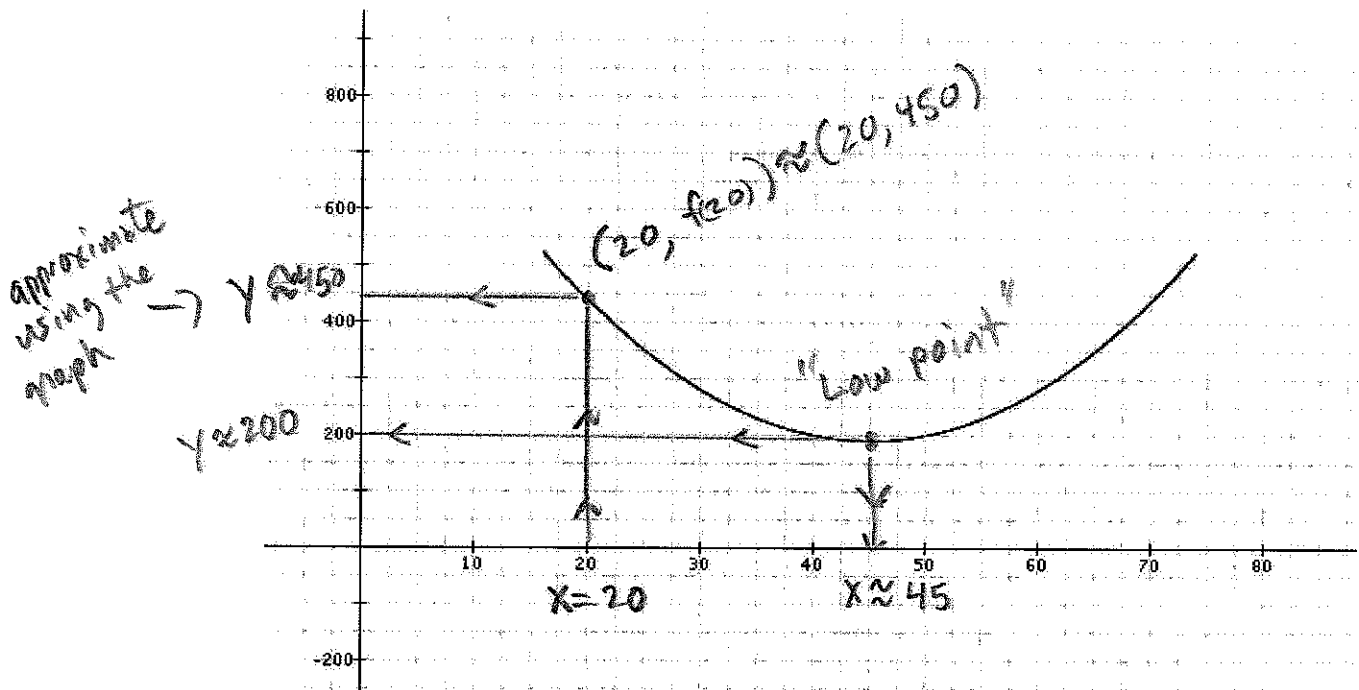
← using algebra

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Example 6: The function

$$f(x) = 0.4x^2 - 36x + 1000$$

models the number of accidents, $f(x)$, per 50 million miles driven as a function of the driver's age, x , in years, where x includes drivers from ages 16 through 74. Use the graph of f , shown below, to answer the questions in a, b and c.



approximate using the graph →

a. Find and interpret $f(20)$.

$$x=20, y=f(20)$$

$$y \approx 450$$

Drivers aged 20 years have about 450 accidents per 50 million miles driven.

b. For what value of x does the graph reach its lowest point?

Use the equation of f to find the minimum value. Interpret this information in the context of the problem.

The x-coordinate of "low point" is approximately 45, or $x \approx 45$.

Drivers aged 45 years have the fewest accidents per 50 million miles driven.

c. Estimate the minimum value from the graph.

The y-coordinate of the "low point" is approximately 200, $y \approx 200$, and $f(45) \approx 200$.

Interval Notation

Domains and ranges are intervals, and interval can be expressed in interval notation, set-builder notation (using inequalities) or graphically on the number line. The following chart shows the different notations. On tests and your homework, you may use interval notation, inequality notation or set-builder notation to depict intervals. **WebAssign often requires interval notation.**

Let a and b represent two real numbers with $a < b$.

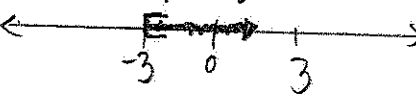
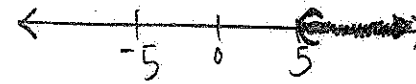
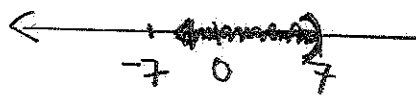
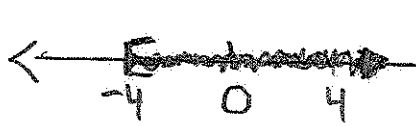
Type of Interval	Interval Notation	Set-Builder Notation	Graph on the Number Line
Closed Interval	$[a,b]$	$\{x \mid a \leq x \leq b\}$	
Open Interval	(a,b)	$\{x \mid a < x < b\}$	
Half-Open Interval	$(a,b]$	$\{x \mid a < x \leq b\}$	
Half-Open Interval	$[a,b)$	$\{x \mid a \leq x < b\}$	
Interval That Is Not Bounded on the Right	$[a,\infty)$	$\{x \mid a \leq x < \infty\}$ or $\{x \mid x \geq a\}$	
Interval That Is Not Bounded on the Right	(a,∞)	$\{x \mid a < x < \infty\}$ or $\{x \mid x > a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,a]$	$\{x \mid -\infty < x \leq a\}$ or $\{x \mid x \leq a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,a)$	$\{x \mid -\infty < x < a\}$ or $\{x \mid x < a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,\infty)$	$\{x \mid -\infty < x < \infty\}$ or $\{x \mid x \text{ is a real no.}\}$	

TRY
Three Examples
From [2,7]

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Review From chapter 2, section 2.7

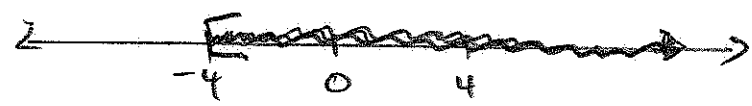

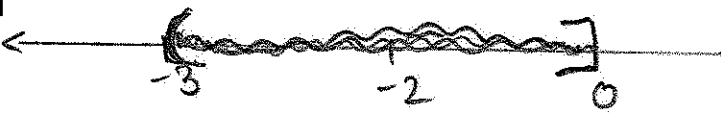
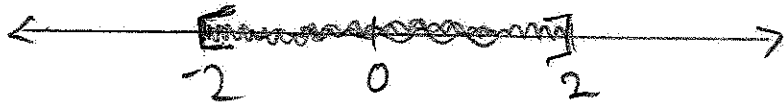
Example 1: Write each inequality in interval notation.

- a. $x \geq -3$  ; $[-3, \infty)$
- b. $5 < x < \infty$  ; $(5, \infty)$
- c. $x < 7$  ; $(-\infty, 7)$
- d. $-4 \leq x < \infty$  ; $[-4, \infty)$

Example 2: Write each interval in set-builder notation.

- a. $[-4, \infty) = \{x \mid -4 \leq x < \infty\}$ or $\{x \mid x \geq -4\}$
- b. $(-\infty, 5) = \{z \mid -\infty < z < 5\}$ or $\{z \mid z < 5\}$
- c. $(-7, -2] = \{k \mid -7 < k \leq -2\}$
- d. $(-1, 4) = \{j \mid -1 < j < 4\}$

Example 3: Graph each interval on the number line.

- a. $[-4, \infty)$ 
- b. $(-\infty, 5)$ 
- c. $(-3, -2]$ 
- e. $[-2, 2]$ 

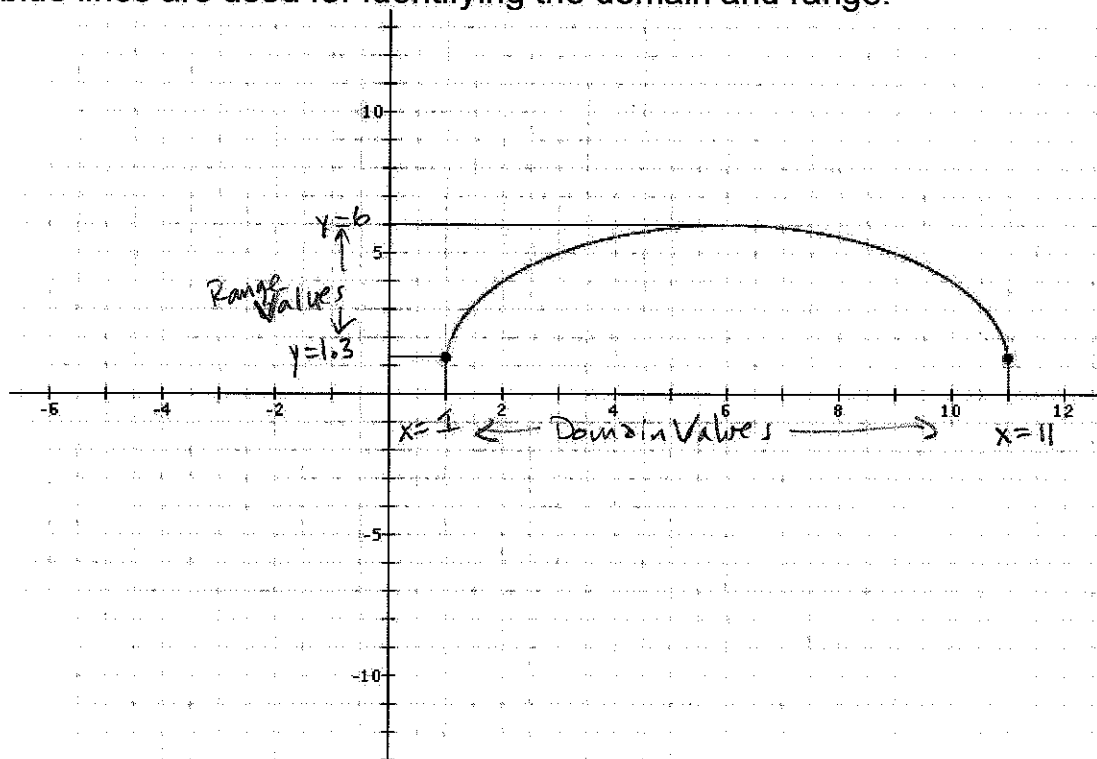
Identifying Domain and Range of a Function from its Graph

To determine the domain and range of a function from its graph:

- Domain: Name the x-axis interval(s) that are traced out when you sweep an imaginary point along the graph and look at the x-coordinates of the points you are tracing.
- Range: Name the y-axis interval(s) that are traced out when you sweep an imaginary point along the graph and look at the y-coordinates of the points you are tracing.

Example 7: Use the graph of each function to identify its domain and range. Give your answers in inequality notation.

a. Note: The graph of the function is given in green. The red and blue lines are used for identifying the domain and range.



Domain: $1 \leq x \leq 11$

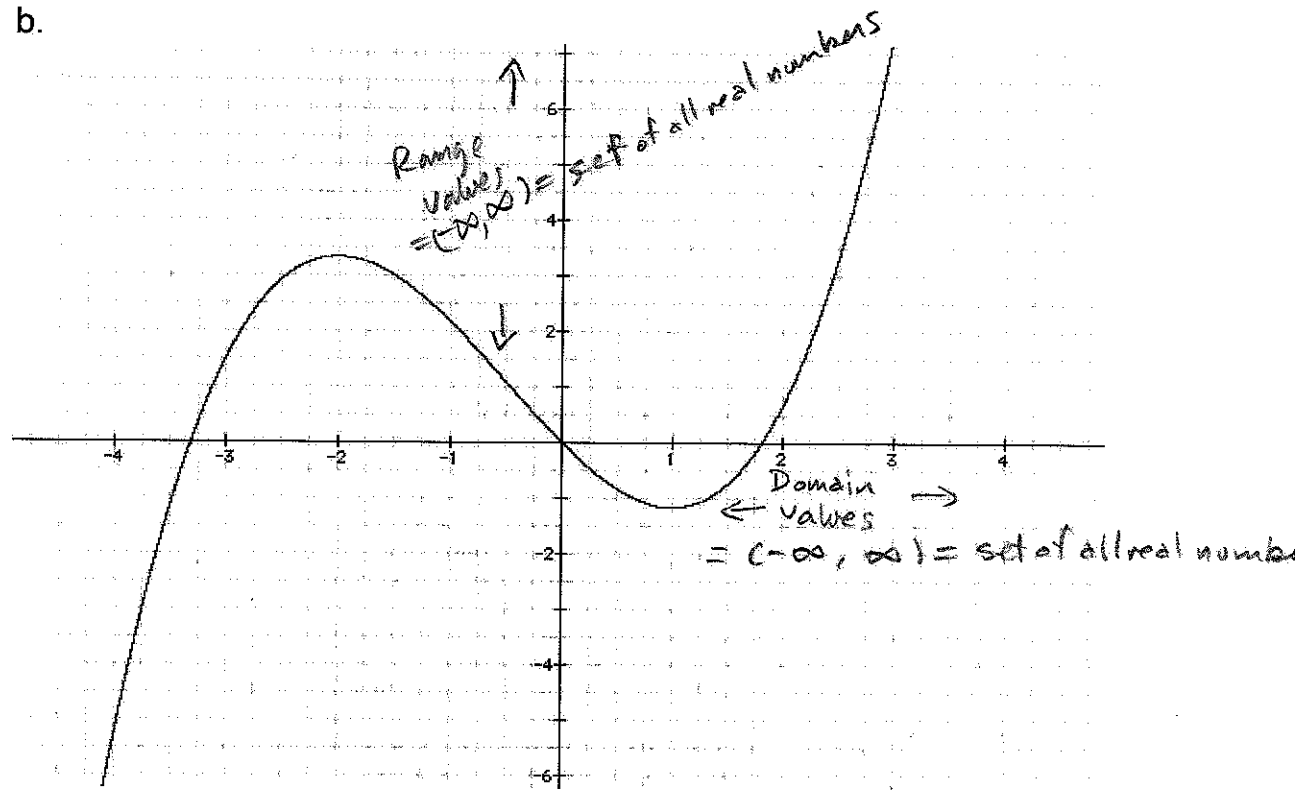
Range: $1.3 \leq y \leq 6$

OR in interval notation: Domain: $[1, 11] = \{x \mid 1 \leq x \leq 11\}$

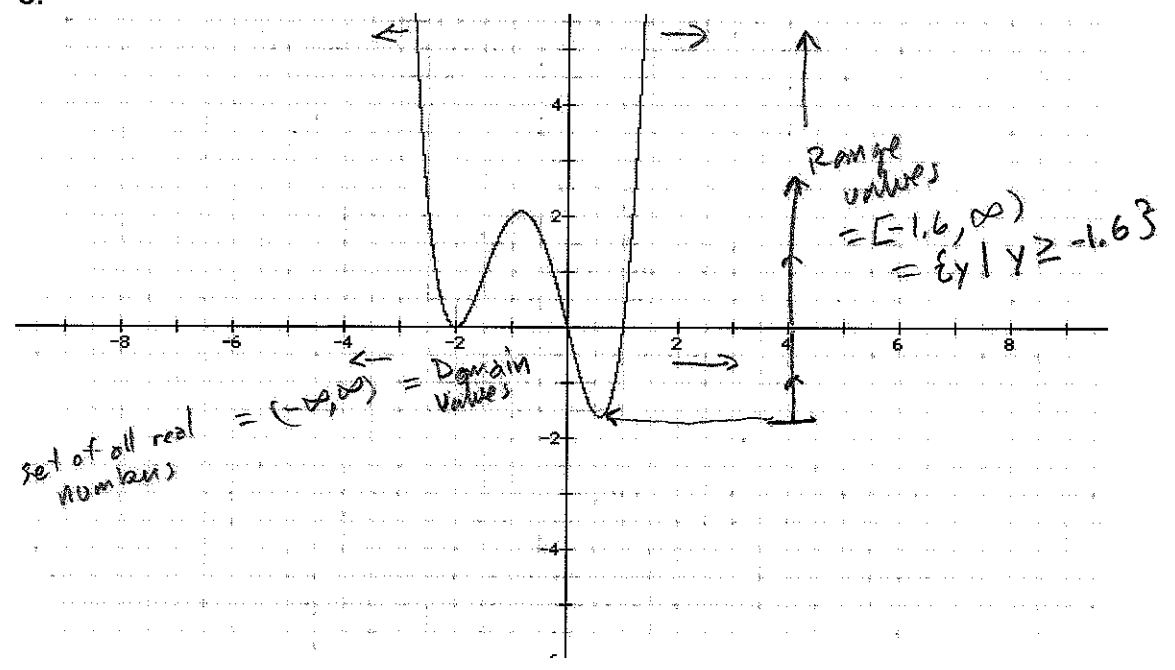
Range: $[1.3, 6] = \{y \mid 1.3 \leq y \leq 6\}$

Ex 7:

b.



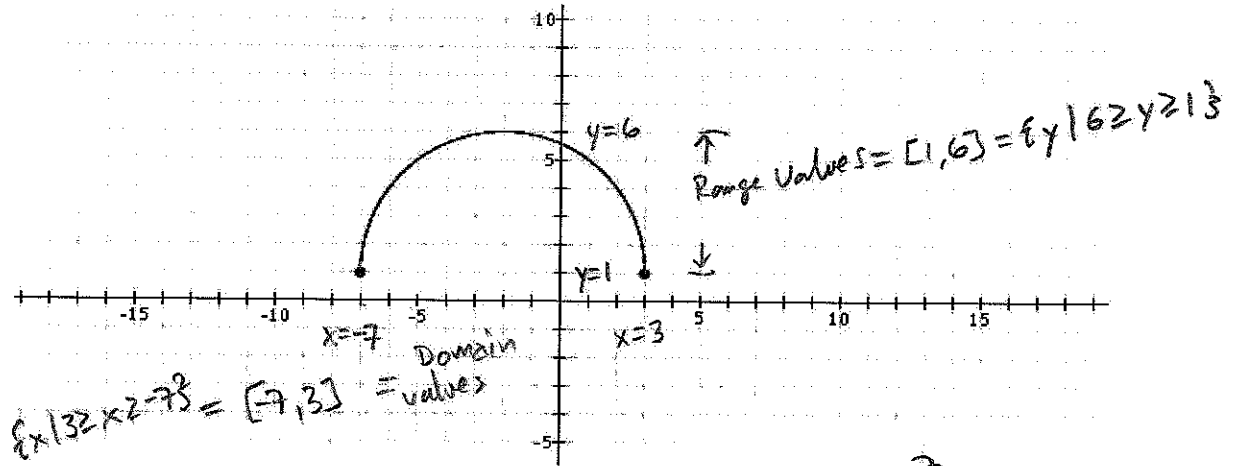
c.



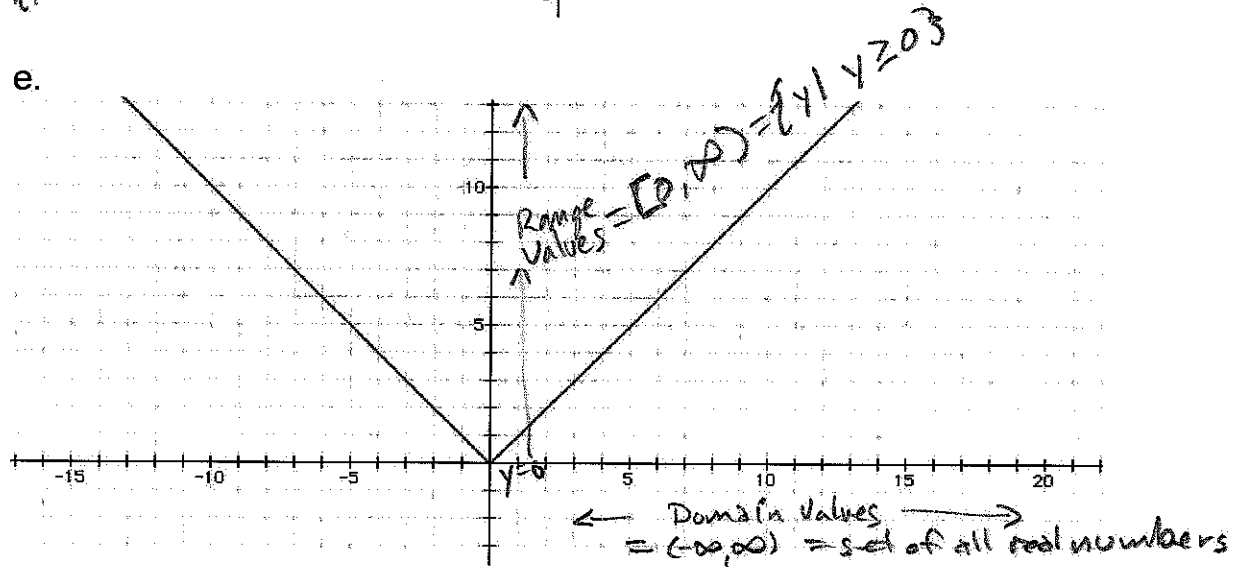
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Ex 7:

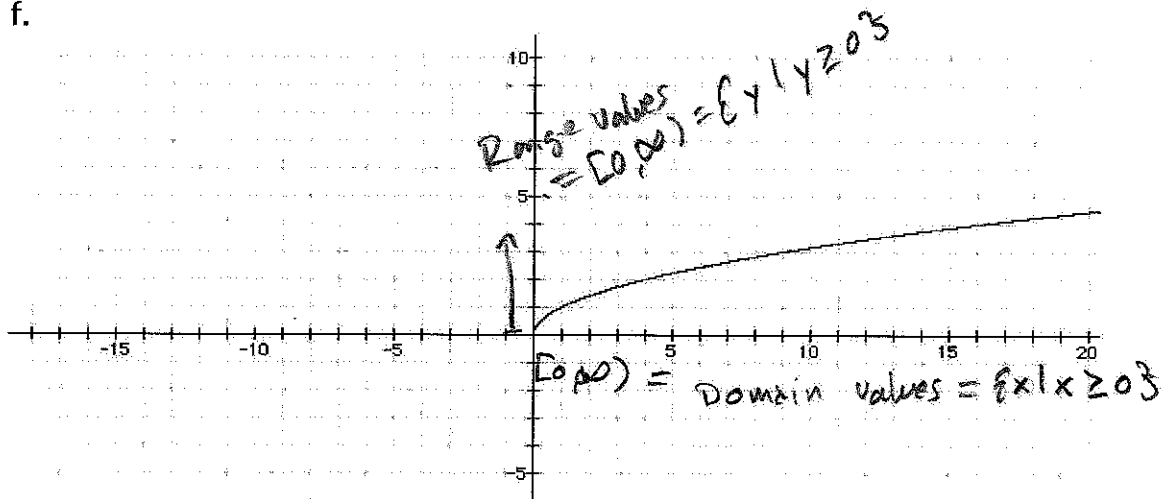
d.



e.



f.



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Answers Section 8.1

Example 1: Domain: {2002, 2003, 2004, 2005}
 Range: {364.2, 424.9, 559.7, 607.1}

Example 2:

- Is a function. Each first element corresponds to exactly one second element.
- Is not a function. The first element "2" corresponds to two different second elements, "6" and "5".
- Is a function. Each first element corresponds to exactly one second element.

Example 3:

- Y is a function of x. The elk population (y) after x years have elapsed must be unique. It isn't logical to think that at the end of, say, 5 elapsed years there would be two different numbers for the elk population. Thus each x (number of years elapsed) must correspond to a unique y (elk population at the end of x years).

$$b. f(x) = \frac{250(3x+5)}{x+25}$$

- $f(3) = 125$. After three years have elapsed, the elk population is 125.

~~Answers Section 8.2~~

Example 4:

- $f(2) = 13$
- $f(-1) = 9$
- $f(q) = \frac{q}{q^2+1}$
- $f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 2$

Example 5:

- Graph represents a function.
- Graph does not represent a function.
- Graph represents a function.
- Graph represents a function.
- Graph does not represent a function.
- Graph does not represent a function.

Example 6:

- $f(20) = 450$ Drivers aged 20 have about 450 accidents per 50 million miles driven.
- The graph reaches its lowest point for the x -value of 45. Using the function $f(45) = 190$. Drivers aged 45 have the lowest number of accidents per 50 million miles driven.
- Estimating from the graph, the minimum value (y value of the lowest point) is about 200.

Example 7:

- Domain: $\{x \mid 1 \leq x \leq 11\}$ (or in interval notation: $[1, 11]$)
Range: $\{y \mid 1.3 \leq y \leq 6\}$ (or in interval notation: $[1.3, 6]$)
- Domain: All real numbers (or in interval notation: $(-\infty, \infty)$)
Range: All real numbers (or in interval notation: $(-\infty, \infty)$)
- Domain: All real numbers (or in interval notation: $(-\infty, \infty)$)
Range: $\{y \mid -1.6 \leq y < \infty\}$ (or in interval notation: $[-1.6, \infty)$)
- Domain: $\{x \mid -7 \leq x \leq 3\}$ (or in interval notation: $[-7, 3]$)
Range: $\{y \mid 1 \leq y \leq 6\}$ (or in interval notation: $[1, 6]$)
- Domain: All real numbers (or in interval notation: $(-\infty, \infty)$)
Range: $\{y \mid 0 \leq y < \infty\}$ (or in interval notation: $[0, \infty)$)
- Domain: $\{x \mid 0 \leq x < \infty\}$ (or in interval notation: $[0, \infty)$)
Range: $\{y \mid 0 \leq y < \infty\}$ (or in interval notation: $[0, \infty)$)

8.3 The Algebra of Functions

The Domain of a Function

Functions that model data often have their domains given, either on the horizontal axis of the graph of the function or along with the function's equation. However, for most functions, only an equation is given and the domain is not specified. In cases like this, the domain is the largest set of numbers for which the value of $f(x)$ is a real number.

Finding the Domain of a Function: If a function f is given by an equation and the domain is not given, find the domain by choosing all real numbers except:

- Any x -value that makes a denominator equal to zero,
- Any x -value that results in a negative number under a square root (or any other even root)
- Any x -value that makes the argument of a logarithmic function negative or zero (We will study these functions in chapter 12).

Example 1: Find the domain of each function:

a. $f(x) = x + 7$; no restrictions ; Domain = set of all real numbers $= (-\infty, \infty)$

b. $f(x) = \sqrt{x-1}$; solve! $x-1 \geq 0$; Domain = $\{x \mid x \geq 1\} = [1, \infty)$

Non-negative numbers in a square root \rightarrow

$1 + x - 1 \geq 0$
 $x \geq 1$



c. $f(x) = \frac{x}{x^2-1}$; solve! $x^2-1=0$

Non-zero denominators \rightarrow

$(x+1)(x-1)=0$
Either $x+1=0$, or $x-1=0$
 $x=-1$, or $x=1$

Domain = $\{x \mid x \neq -1 \text{ and } x \neq 1\}$

d. $f(x) = \sqrt{2x-7}$ solve!

Non-negative numbers in a square root \rightarrow

$2x-7 \geq 0$
 $7+2x-7 \geq 7+0$
 $2x \geq 7$

$\frac{2x}{2} \geq \frac{7}{2}$; Domain = $\{x \mid x \geq \frac{7}{2}\} = [\frac{7}{2}, \infty)$
 $x \geq \frac{7}{2}$

e. $f(x) = \frac{2x-4}{x^2-2x-3}$

Non-zero denominators \rightarrow

solve! $x^2-2x-3=0$
 $(x-3)(x+1)=0$
Either $x-3=0$, or $x+1=0$
 $x=3$, or $x=-1$

Domain = $\{x \mid x \neq 3 \text{ and } x \neq -1\}$

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The Algebra of Functions

Two functions can be added, subtracted, multiplied or divided as long as there are numbers common to the domains of both functions. The common domain for the sum, difference, product or quotient of two functions is the set of numbers that are common to the domains of both functions.

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The sum $f + g$, the difference $f - g$, the product fg , and the quotient $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g . They are defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

Example 2: Let $f(x) = 2x + 1$ and $g(x) = x^2 - x$. Find:

$$\begin{aligned} \text{a. } (f + g)(2) &= f(2) + g(2) && = (5) + (2) \\ &= [2(2) + 1] + [(2)^2 - (2)] && = 7 \quad ; \quad \underline{(f + g)(2) = 7} \\ &= (4 + 1) + (4 - 2) \end{aligned}$$

$$\begin{aligned} \text{b. } (f - g)(-1) &= f(-1) - g(-1) && = (-1) - (-2) \\ &= [2(-1) + 1] - [(-1)^2 - (-1)] && = -3 \quad ; \quad \underline{(f - g)(-1) = -3} \\ &= [-2 + 1] - [1 + 1] \end{aligned}$$

$$\begin{aligned} \text{c. } (f - g)(x) &= f(x) - g(x) && = -x^2 + 3x + 1 \quad ; \quad \underline{(f - g)(x) = -x^2 + 3x + 1} \\ &= (2x + 1) - (x^2 - x) \\ &= 2x + 1 - x^2 + x \end{aligned}$$

$$\begin{aligned} \text{d. } (fg)(3) &= [f(3)] \cdot [g(3)] && = [6 + 1] \cdot [9 - 3] \\ &= [2(3) + 1] \cdot [(3)^2 - (3)] && = [7] \cdot [6] \\ &&& = 42 \quad ; \quad \underline{(fg)(3) = 42} \end{aligned}$$

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Answers Section 8.3

Example 1:

- a. Domain: All real numbers
- b. Domain: $\{x/ x \text{ is a real number and } x \geq 1\}$
- c. Domain: $\{x/ x \text{ is a real number and } x \neq 1 \text{ and } x \neq -1\}$
- d. Domain: $\{x/ x \text{ is a real number and } x \geq \frac{7}{2}\}$
- e. Domain: $\{x/ x \text{ is a real number and } x \neq 3 \text{ and } x \neq -1\}$

Example 2:

- a. $(f + g)(2) = 7$
- b. $(f - g)(-1) = -3$
- c. $(f - g)(x) = -x^2 + 3x + 1$
- d. $(fg)(3) = 42$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(3x-4) \\
 &= 5(3x-4)+2 \\
 &= 15x-20+2 \\
 &= 15x-18
 \end{aligned}$$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g(5x+2) \\
 &= 3(5x+2)-4 \\
 &= 15x+6-4 \\
 &= 15x+2
 \end{aligned}$$

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8.4 Composite and Inverse Functions

The Composition of Functions

The composition of the function f with g is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is the set of all x such that

- x is in the domain of g and
- $g(x)$ is in the domain of f .

Forming Composite Functions

Example 1: Given $f(x) = 5x + 2$ and $g(x) = 3x - 4$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= 5 \cdot (g(x)) + 2 \\
 &= 5(3x-4) + 2 \\
 &= 15x - 20 + 2 \\
 &= 15x - 18
 \end{aligned}$$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= 3(f(x)) - 4 \\
 &= 3(5x+2) - 4 \\
 &= 15x + 6 - 4 \\
 &= 15x + 2
 \end{aligned}$$

$$(f \circ g)(x) = 15x - 18$$

$$(g \circ f)(x) = 15x + 2$$

Inverse Functions

Let f and g be two functions such that

$$f(g(x)) = x \text{ for every } x \text{ in the domain of } g, \text{ and}$$

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f.$$

The function g is the inverse of the function f , and is denoted by f^{-1} (read "f-inverse"). Thus $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1} and vice versa.

Verifying Inverse Functions

Example 2: Verify that each function is the inverse of the other:

$$f(x) = 6x \text{ and } g(x) = \frac{x}{6}$$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= 6 \cdot [g(x)] \\
 &= \frac{6}{1} \cdot \left(\frac{x}{6}\right)
 \end{aligned}$$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= \frac{[f(x)]}{6} \\
 &= \frac{6x}{6}
 \end{aligned}$$

$$(f \circ g)(x) = x$$

$$(g \circ f)(x) = x$$

AND

Since $(f \circ g)(x) = (g \circ f)(x) = x$, we can say that f and g are inverses.

Example 3: Verify that each function is the inverse of the other.

$$f(x) = 4x + 9 \text{ and } g(x) = \frac{x-9}{4}$$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= 4 \cdot [g(x)] + 9 \\ &= \frac{4 \cdot [x-9]}{4} + 9 \\ &= x - 9 + 9 \end{aligned}$$

$$\underline{(f \circ g)(x) = x}$$

$$\begin{aligned} \text{AND } (g \circ f)(x) &= g(f(x)) \\ &= \frac{[f(x)] - 9}{4} \\ &= \frac{[4x+9] - 9}{4} \\ &= \frac{4x}{4} \\ (g \circ f)(x) &= x \end{aligned}$$

Since $(f \circ g)(x) = (g \circ f)(x) = x$, we can say that f and g are inverses.

Finding the Inverse of a Function

The equation for the inverse of a function can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y .
3. Solve for y . If this equation does not define y as a function of x , the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x , the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 with $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example 4: Find the inverse of $f(x) = 6x + 3$

① let $y = 6x + 3$

② $x = 6y + 3$

③ Solve for y : $x = 6y + 3$

$$\begin{aligned} -3 + x &= -3 + 6y + 3 \\ x - 3 &= 6y \\ \frac{x-3}{6} &= \frac{6y}{6} \\ \frac{x-3}{6} &= y \end{aligned}$$

④ $f^{-1}(x) = \frac{x-3}{6}$

Verify: $f(f^{-1}(x)) = x = f^{-1}(f(x))$

$$f(f^{-1}(x)) = 6[f^{-1}(x)] + 3$$

$$f(f^{-1}(x)) = 6\left[\frac{x-3}{6}\right] + 3$$

$$f(f^{-1}(x)) = x - 3 + 3$$

$$f(f^{-1}(x)) = x \quad \checkmark$$

$$f^{-1}(f(x)) = \frac{[f(x)] - 3}{6}$$

$$f^{-1}(f(x)) = \frac{[6x+3] - 3}{6}$$

$$f^{-1}(f(x)) = \frac{6x}{6}$$

$$f^{-1}(f(x)) = x \quad \checkmark$$

④ $f^{-1}(x) = \sqrt[3]{x-1}$

Verify!

$$f(f^{-1}(x)) = [f^{-1}(x)+1]^3$$

$$= [\sqrt[3]{x-1}+1]^3$$

$$= [\sqrt[3]{x}]^3$$

$$= x \quad \checkmark$$

$$f^{-1}(f(x)) = \sqrt[3]{f(x)} - 1$$

$$= \sqrt[3]{(x+1)^3} - 1$$

$$= (x+1) - 1$$

$$= x \quad \checkmark$$

Example 5: Find the inverse of $f(x) = (x+1)^3$

① let $y = (x+1)^3$

② $x = (y+1)^3$

③ Solve for y! $x = (y+1)^3$

$$\sqrt[3]{x} = y+1$$

$$-1 + \sqrt[3]{x} = -1 + y+1$$

$\rightarrow \sqrt[3]{x}-1 = y$

Example 6: Find the inverse of $f(x) = x^3 - 4$

① let $y = x^3 - 4$

② $x = y^3 - 4$

③ Solve for y! $x = y^3 - 4$

$$x+4 = y^3 - 4 + 4$$

$$x+4 = y^3$$

$\rightarrow \sqrt[3]{x+4} = y$

④ $f^{-1}(x) = \sqrt[3]{x+4}$

Verify! $f(f^{-1}(x)) = [f^{-1}(x)]^3 - 4 = [\sqrt[3]{x+4}]^3 - 4$

$$f(f^{-1}(x)) = (x+4) - 4 = x \quad \checkmark$$

$$f^{-1}(f(x)) = \sqrt[3]{f(x)+4} = \sqrt[3]{(x^3-4)+4}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3} = x \quad \checkmark$$

The Horizontal Line Test and One-to-One Functions

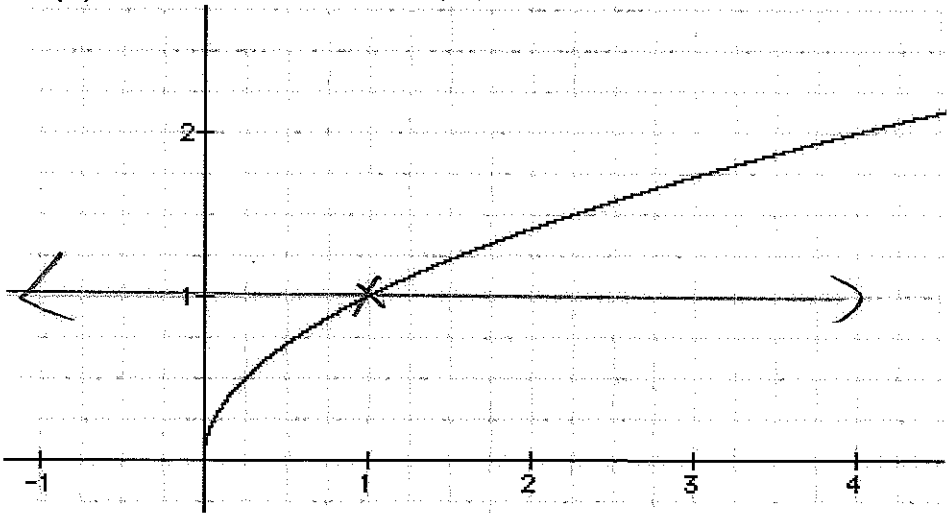
The Horizontal Line Test for Functions

A function f has an inverse that is a function, f^{-1} , if there is no horizontal line that intersects the graph of the function f at more than one point.

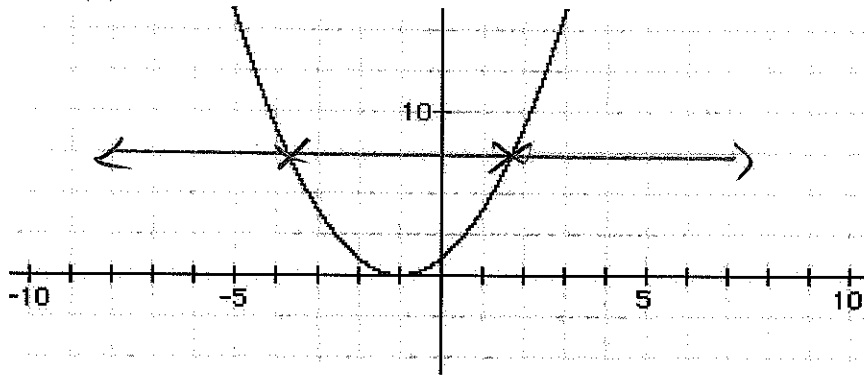
Example 7: For each of the following functions, use the given graph of the function and the horizontal line test to determine if the function has an inverse.

a. $f(x) = \sqrt{x}$

Passes Horizontal Line Test; This function has an inverse.

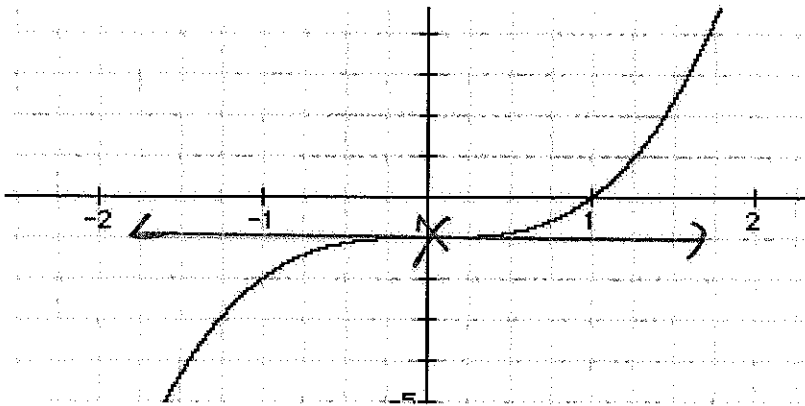


b. $f(x) = x^2 + 2x + 1$



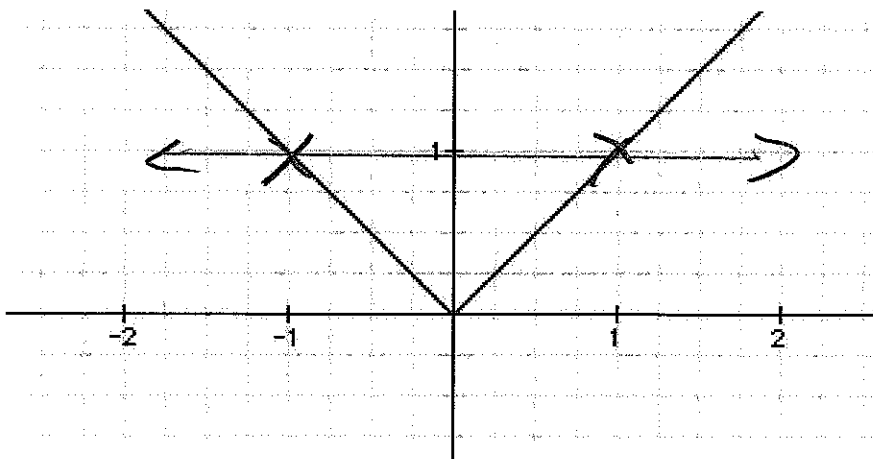
Fails Horizontal Line Test;
 $f(x)$ does not have
 an inverse function

c. $f(x) = x^3 - 1$



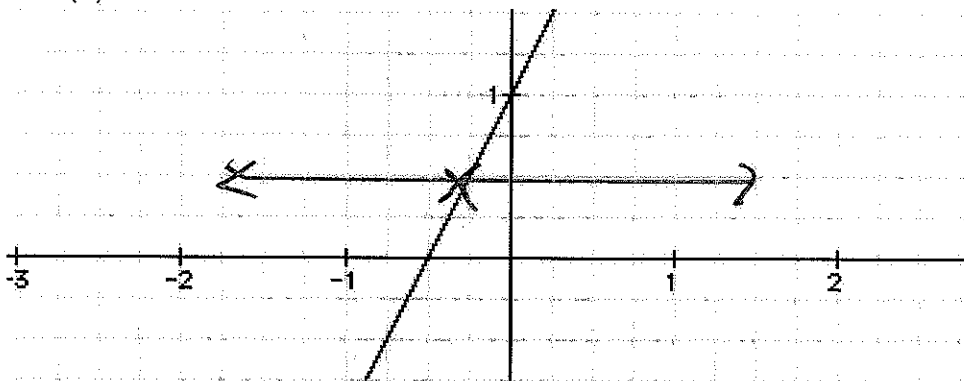
Passes Horizontal Line Test; $f(x)$ does have
 an inverse function

d. $f(x) = |x|$



Fails Horizontal Line Test; $f(x)$ does not
 have an inverse
 function

e. $f(x) = 2x + 1$ Passes Horizontal Line Test; $f(x)$ does have an inverse function

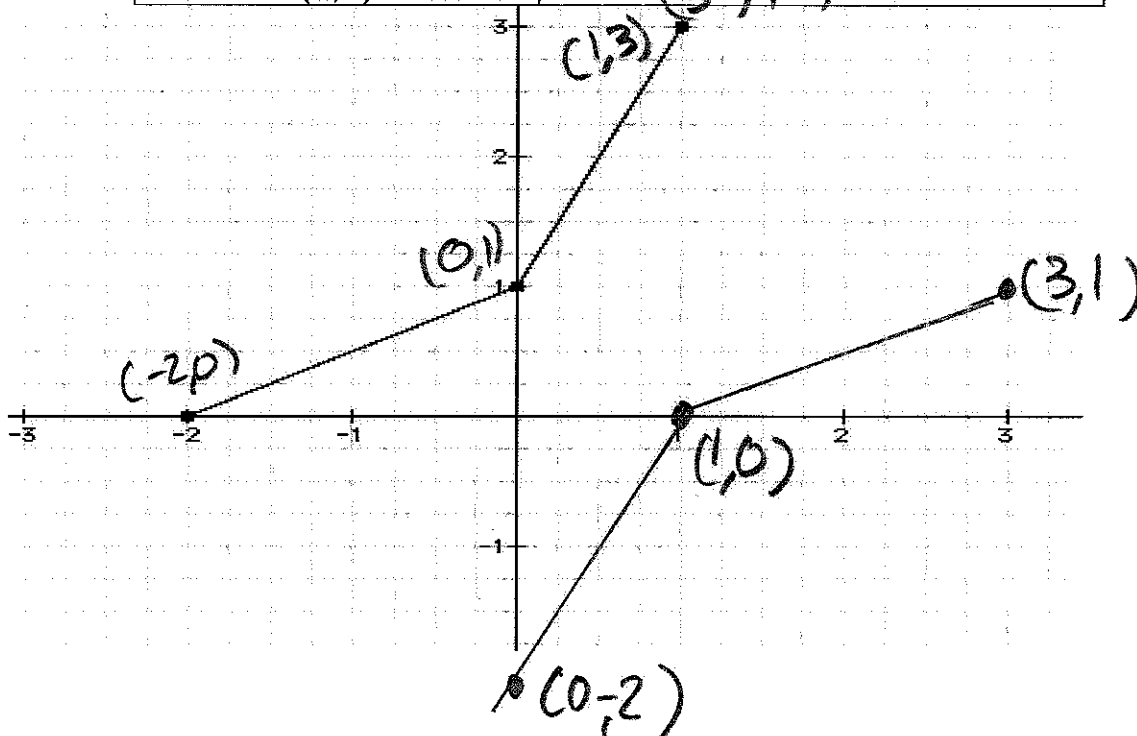


Graphs of f and f^{-1}

The graphs of f and f^{-1} are reflections of one another through the line $y = x$. Points on the graph of f^{-1} can be found by reversing the coordinates of the points on the graph of f .

Example 8: Consider the graph of the function f traced by joining the points given below with straight-line segments. Sketch the graph of f and the graph of f^{-1} .

Points on $y = f(x)$	Points on $y = f^{-1}(x)$
$(-2, 0)$	$(0, -2)$
$(0, 1)$	$(1, 0)$
$(1, 3)$	$(3, 1)$



Answers Section 8.4

Example 1: $(f \circ g)(x) = 15x - 18$ and $(g \circ f)(x) = 15x + 2$

Example 2: $(f \circ g)(x) = (g \circ f)(x) = x$ f and g are inverses of one another.

Example 3: $(f \circ g)(x) = (g \circ f)(x) = x$ f and g are inverses of one another.

Example 4: $f^{-1}(x) = \frac{x-3}{6}$

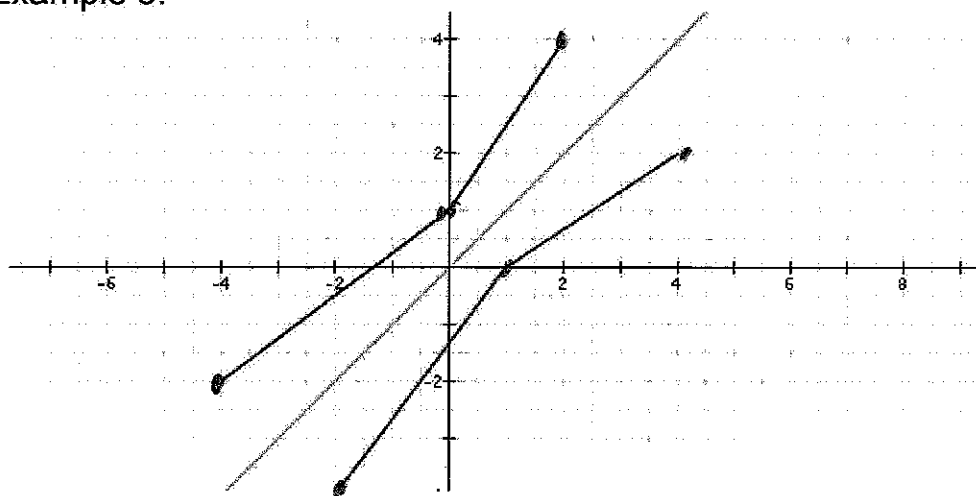
Example 5: $f^{-1}(x) = \sqrt[3]{x} - 1$

Example 6: $f^{-1}(x) = \sqrt[3]{x+4}$

Example 7:

- The graph passes the horizontal line test, and thus the function graphed has an inverse function.
- The graph fails the horizontal line test, and thus the function graphed does not have an inverse function.
- The graph passes the horizontal line test, and thus the function graphed has an inverse function.
- The graph fails the horizontal line test, and thus the function graphed does not have an inverse function.
- The graph passes the horizontal line test, and thus the function graphed has an inverse function.

Example 8:



Section 9.1 Solving Linear Inequalities

We know that a linear equation in x can be expressed as $ax + b = 0$. A **linear inequality in x** can be written in one of the following forms: $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, or $ax + b \geq 0$. In each form, $a \neq 0$.

If an inequality does not contain fractions, it can be solved using the following procedure. Notice how similar this procedure is to the procedure for solving a linear equation.

Steps for solving a linear inequality

- Step 1. Simplify each side.
- Step 2. Collect variable terms on one side and constant terms on the other (*use addition property of inequalities*)
- Step 3. Isolate the variable and solve (*use multiplication property of inequalities, change the sense of the inequality when multiplying, or dividing both sides by a negative number*)
- Step 4. Express the solution set in interval notation or set-builder notation and graph the solution set on a number line.

Example 1: Solve and graph the solution set on a number line:

$$3x - 5 > -17$$

$$3x - 5 + 5 > -17 + 5$$

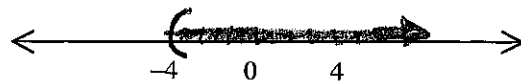
$$3x > -12$$

$$\frac{1}{3} \cdot \frac{3x}{1} > \frac{-12}{1} \cdot \frac{1}{3}$$

$$x > -4$$

The solution set is

$$\{x \mid x > -4\} = (-4, \infty).$$



Example 2: Solve and graph the solution set on a number line:

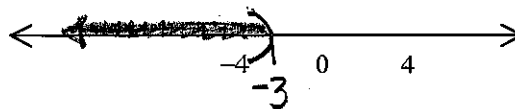
$$\begin{aligned} -2x - 4 &> x + 5 \\ -2x - 4 + 2x &> 2x + x + 5 \end{aligned}$$

$$\begin{aligned} -4 &> 3x + 5 \\ -5 + (-4) &> -5 + 3x + 5 \end{aligned}$$

$$-9 > 3x$$

$$-\frac{9}{3} > \frac{3x}{3}$$

$$-3 > x$$



The solution set is $\{x \mid x < -3\} = (-\infty, -3)$.

If an inequality contains fractions, begin by multiplying both sides by the least common denominator. This will clear the inequality of fractions.

LCD=6 Example 3: Solve and graph the solution set on a number line:

$$\frac{x-4}{2} \geq \frac{x-2}{3} + \frac{5}{6}$$

$$\frac{6}{1} \cdot \left[\frac{x-4}{2} \right] \geq \frac{6}{1} \cdot \left[\frac{x-2}{3} + \frac{5}{6} \right]$$

$$3(x-4) \geq \frac{6}{1} \cdot \left[\frac{x-2}{3} \right] + \frac{6}{1} \cdot \frac{5}{6}$$

$$3x - 12 \geq 2(x-2) + 5$$

$$3x - 12 \geq 2x - 4 + 5$$

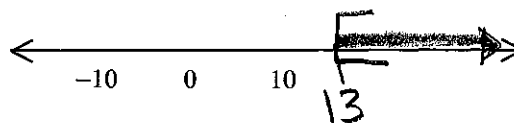
$$3x - 12 \geq 2x + 1$$

$$-2x + 3x - 12 \geq -2x + 2x + 1$$

$$x - 12 \geq 1$$

$$12 + x - 12 \geq 12 + 1$$

$$x \geq 13$$



The solution set is $\{x \mid x \geq 13\} = [13, \infty)$.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 4: You are choosing between two telephone plans. Plan A has a monthly fee of \$15 with a charge of 8 cents per minute for all calls. Plan B has a monthly fee of \$3 with a charge of 12 cents per minute for all calls. How many minutes of calls in a month make plan A the better deal? (Define a variable, create an inequality, solving using algebra, and answer in a sentence.) Let x = minutes used in the call plan

$$\text{Monthly Cost of Plan A} = \$15 + \$0.08x$$

$$\text{Monthly Cost of Plan B} = \$3 + \$0.12x$$

Plan A is a better deal when it costs less than plan B.

$$15 + 0.08x < 3 + 0.12x$$

$$-0.08x + 15 + 0.08x < -0.08x + 3 + 0.12x$$

$$15 < 3 + 0.04x$$

$$-3 + 15 < -3 + 3 + 0.04x$$

$$12 < 0.04x$$

$$\frac{100}{4} \cdot \frac{12}{1} < \frac{100}{4} \cdot \frac{4}{100} x$$

$$300 < x$$

ANS: Plan A is a better deal when more than 300 monthly call minutes are used.

SDWK
$0.04 = \frac{4}{100}$
$\frac{100}{4} \cdot \frac{12}{1} = 100.3$
$= 300$

APPLICATION: For a business to realize a profit, the revenue (or income), R , must be greater than the Cost, C . That is, a profit will be obtained only when $R > C$. The company breaks even when $R = C$.

If you sell x units of a product at a certain price p , then your revenue function is $R(x) = \underline{pX}$.

The cost of your business may include a fixed cost (like rental fees, initial cost of equipment, etc.) and the cost of making each item.

$$C(x) = \text{fixed cost} + \left(\frac{\text{Cost per unit}}{\text{Produced}} \right) \cdot X$$

The profit $P(x)$, generated after producing and selling x units of a product is given by the profit function:

$$P(x) = \underline{R(x)} - \underline{C(x)}$$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 5: Technology is now promising to bring light, fast, and beautiful wheelchairs to millions of disabled people. A company is planning to manufacture these radically different wheelchairs. Fixed cost will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will be sold for \$600.

a) Write the cost function, C , of producing x wheelchairs.

$$C(x) = \$500,000 + \$400x$$

b) Write the revenue function, R , of producing x wheelchairs.

$$R(x) = \$600x$$

c) Write the profit function, P , from producing and selling x wheelchairs.

$$P(x) = R(x) - C(x)$$

$$\underline{P(x) = 200x - 500,000}$$

$$P(x) = 600x - (500,000 + 400x)$$

d) How many wheelchairs must be produced and sold for the business to make money?

To make money, the profit, $P(x)$, must be positive. That is, $P(x) > 0$.

Solve:

$$P(x) > 0$$

$$200x - 500,000 > 0$$

$$200x - 500,000 + 500,000 > 500,000 + 0$$

$$200x > 500,000$$

$$\frac{200x}{200} > \frac{500,000}{200}$$

$$x > 2,500$$

500k
$\frac{500,000}{200}$
$= \frac{5000}{2}$
$= \frac{2 \cdot 2,500}{2}$
$= 2,500$

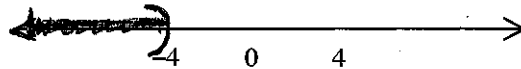
ANS: More than 2,500 wheelchairs must be produced and sold for the business to make money.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Extra Practice: Solve the given inequalities and graph the solution set. Express your answer in interval notation.

Example 6:

a. $-4(x+2) > 3x+20$



$$\begin{aligned} -4x - 8 &> 3x + 20 \\ -4x - 8 + 4x &> 4x + 3x + 20 \\ -8 &> 7x + 20 \\ -20 + (-8) &> -20 + 7x + 20 \\ -28 &> 7x \end{aligned}$$

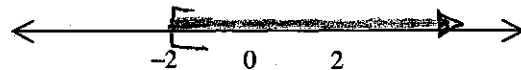
$$\frac{-28}{7} > \frac{7x}{7}$$

$$-4 > x$$

The solution set is $\{x \mid x < -4\} = (-\infty, -4)$.

LCD=10

b. $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$



$$\begin{aligned} \frac{10}{1} \cdot \left[\frac{3x}{10} + 1 \right] &\geq \frac{10}{1} \cdot \left[\frac{1}{5} - \frac{x}{10} \right] \\ \frac{10}{1} \cdot \frac{3x}{10} + \frac{10}{1} \cdot 1 &\geq \frac{10}{1} \cdot \frac{1}{5} - \frac{10}{1} \cdot \frac{x}{10} \\ 3x + 10 &\geq 2 - x \\ x + 3x + 10 &\geq x + 2 - x \\ 4x + 10 &\geq 2 \end{aligned}$$

$$-10 + 4x + 10 \geq -10 + 2$$

$$4x \geq -8$$

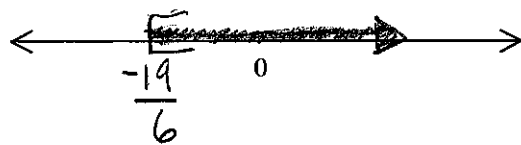
$$\frac{4x}{4} \geq \frac{-8}{4}$$

$$x \geq -2$$

The solution set is $\{x \mid x \geq -2\} = [-2, \infty)$.

LCD=12

c. $\frac{4x-3}{6} \geq \frac{2x-1}{12} - 2$



$$\begin{aligned} \frac{12}{1} \cdot \left[\frac{4x-3}{6} \right] &\geq \frac{12}{1} \cdot \left[\frac{2x-1}{12} - \frac{2}{1} \right] \\ 2(4x-3) &\geq \frac{12}{1} \cdot \left[\frac{2x-1}{12} \right] - \frac{12}{1} \cdot \frac{2}{1} \\ 8x - 6 &\geq 2x - 1 - 24 \\ 8x - 6 &\geq 2x - 25 \\ -2x + 8x - 6 &\geq -2x + 2x - 25 \\ 6x - 6 &\geq -25 \\ 6x - 6 + 6 &\geq -25 + 6 \\ 6x &\geq -19 \end{aligned}$$

$$\frac{6x}{6} \geq \frac{-19}{6}$$

$$x \geq \frac{-19}{6}$$

The solution set is $\{x \mid x \geq \frac{-19}{6}\}$.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Answers Section 9.1

Example 1: $(-4, \infty)$

Example 2: $(-\infty, -3)$

Example 3: $[13, \infty)$

Example 4:

let $x =$ minutes used in call plan, $15 + 0.08 < 3 + 0.12x$, $x > 300$,

Plan A is a better deal when you use more than 300 minutes of calls.

Example 5a: $C(x) = 500,000 + 400x$

Example 5b: $R(x) = 600x$

Example 5c: $P(x) = 200x - 500,000$

$$200x - 500,000 > 0, \quad x > 2,500,$$

Example 5d: More than 2,500 wheelchairs must be produced and sold for the business to make money.

Extra Practice:

Example 6a: $(-\infty, -4)$

Example 6b: $[-2, \infty)$

Example 6c: $\left[-\frac{19}{6}, \infty\right)$

Common Student Error: Students often forget to change the sense, (the direction), of the inequality when multiplying or dividing by a negative number.

Given: $-3x < 6$

$$\frac{-3x}{-3} < \frac{6}{-3} \rightarrow x < -2 \text{ is WRONG } \times$$
$$\frac{-3x}{-3} > \frac{6}{-3} \rightarrow x > -2 \text{ is CORRECT } \checkmark$$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Section 9.2 Compound Inequalities

A **compound inequality** is formed by joining two inequalities with the word AND, or the word OR.

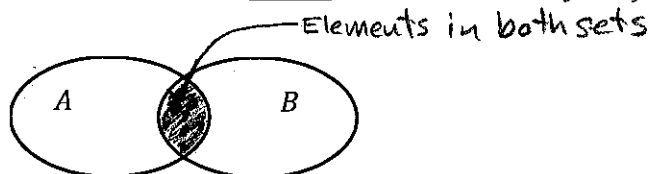
For example: $3 < x$ AND $x < 5$

$$x + 4 > 3 \quad \text{OR} \quad 2x - 3 \leq 6$$

Definition of the intersection of two sets---AND

For any two sets A and B , the intersection of A and B , symbolized $A \cap B$, is defined as follows:

$$A \cap B = \{ x \mid x \text{ is an element of } A \text{ AND } x \text{ is an element of } B \}$$



Example 1: Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$. Find $A \cap B$.

$$A \cap B = \{2, 4\}$$

Example 2: Let $C = \{1, 3, 5, 7\}$ and $D = \{4, 6, 8\}$. Find $C \cap D$.

$$C \cap D = \emptyset \quad \text{"or"} \quad C \cap D = \{ \}$$

"Empty Set"

A number is a **solution of a compound inequality** formed by the word **AND** if it is a solution of both inequalities. Thus, the solution set is the intersection of the solution sets of the two inequalities.

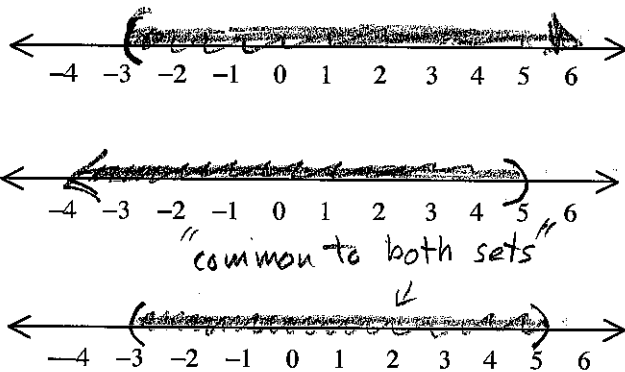
Steps for solving a compound inequalities involving AND

Step 1. Solve each inequality individually.

Step 2. Graph the solution set to each inequality on a number line and take the intersection of these solution sets. This intersection appears as the portion of the number line that the two graphs have in common.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 3: Solve: $-3 < x$ AND $x < 5$



The solution set
 is $\{x \mid -3 < x \text{ and } x < 5\}$
 $= (-3, 5)$
 $= \{x \mid -3 < x < 5\}$

Example 4: Solve: $-3x - 2 > 5$ AND $5x - 1 \leq -21$

$$-3x - 2 > 5$$

$$-3x - 2 + 2 > 5 + 2$$

$$-3x > 7$$

$$\frac{-3x}{-3} < \frac{7}{-3}$$

$$x < -\frac{7}{3}$$

← "Look"

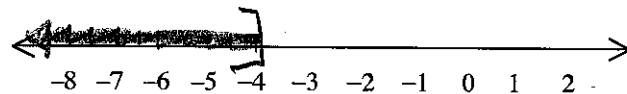
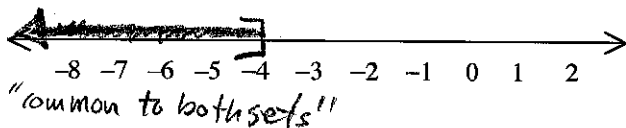
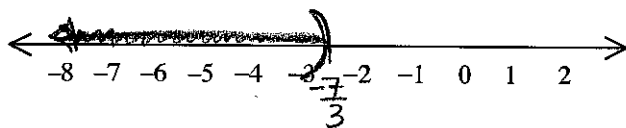
$$5x - 1 \leq -21$$

$$5x - 1 + 1 \leq -21 + 1$$

$$5x \leq -20$$

$$\frac{5x}{5} \leq \frac{-20}{5}$$

$$x \leq -4$$



The solution set
 is $\{x \mid x \leq -4\} = (-\infty, -4]$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 5: Solve: $3x + 2 \leq 11$ AND $-2x - 3 < 5$

$$3x + 2 \leq 11$$

$$3x + 2 - 2 \leq 11 - 2$$

$$3x \leq 9$$

$$\frac{3x}{3} \leq \frac{9}{3}$$

$$x \leq 3$$

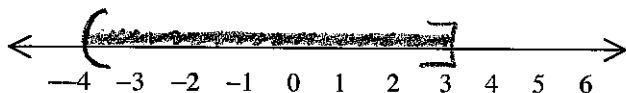
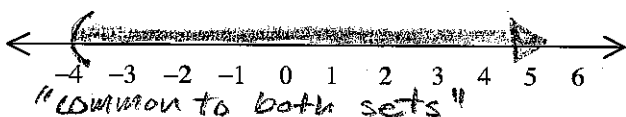
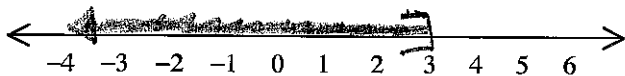
$$-2x - 3 < 5$$

$$-2x - 3 + 3 < 5 + 3$$

$$-2x < 8 \quad \text{"LOOK"}$$

$$\frac{-2x}{-2} > \frac{8}{-2}$$

$$x > -4$$



The solution set
is $\{x \mid -4 < x \leq 3\}$
 $= (-4, 3]$

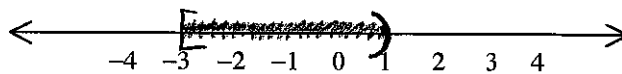
If $a < b$, the compound inequality $a < x$ AND $x < b$ can be written in the shorter form $a < x < b$. For example, the compound inequality $-5 < 2x + 1$ AND $2x + 1 < 3$ can be abbreviated $-5 < 2x + 1 < 3$.

The word *AND* does not appear when the inequality is written in the shorter form, although it is implied. The shorter form enables us to solve both inequalities at once. By performing the same operations on all three parts of the inequality, our goal is to **isolate x in the middle**.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 6: Solve: $-5 \leq 2x + 1 < 3$

$$\begin{aligned} -5 &\leq 2x + 1 < 3 \\ -5 + (-1) &\leq 2x + 1 + (-1) < 3 + (-1) \\ -6 &\leq 2x < 2 \\ \frac{-6}{2} &\leq \frac{2x}{2} < \frac{2}{2} \\ -3 &\leq x < 1 \end{aligned}$$



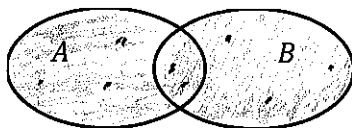
The solution set is
 $\{x \mid -3 \leq x < 1\}$
 $= [-3, 1)$.

Definition of the union of two sets----OR

For any two sets A and B , the union of A and B , symbolized $A \cup B$, is defined as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}$$

"Join the sets together"



Example 7: Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$. Find $A \cup B$.

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Example 8: Let $C = \{1, 3, 5, 7\}$ and $D = \{4, 6, 8\}$. Find $C \cup D$.

$$C \cup D = \{1, 3, 4, 5, 6, 7, 8\}$$

A number is a **solution of a compound inequality** formed by the word **OR** if it is a solution of either inequality. Thus, the solution set of a compound inequality formed by the word **OR** is the union of the solution sets of the two inequalities.

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

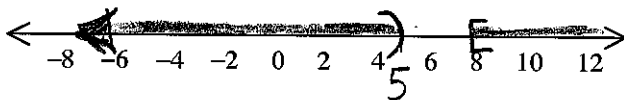
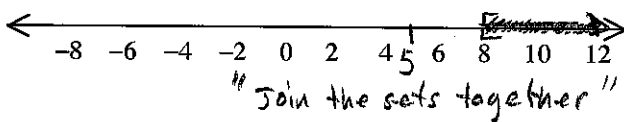
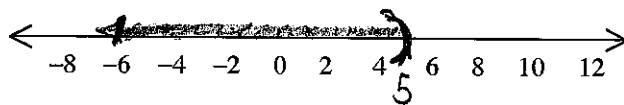
Steps for solving a compound inequalities involving OR

Step 1. Solve each inequality individually.

Step 2. Graph the solution set to each inequality on a number line and take the union of these solution sets. This union appears as the portion of the number line representing the total collection of numbers in the two graphs.

Example 9: Solve: $2x - 3 < 7$ OR $35 - 4x \leq 3$

$$\begin{array}{l}
 2x - 3 < 7 \\
 2x - 3 + 3 < 7 + 3 \\
 2x < 10 \\
 \frac{2x}{2} < \frac{10}{2} \\
 x < 5
 \end{array}
 \quad
 \begin{array}{l}
 35 - 4x \leq 3 \\
 -35 + 35 - 4x \leq -35 + 3 \\
 -4x \leq -32 \quad \text{"LOOK"} \\
 \frac{-4x}{-4} \geq \frac{-32}{-4} \\
 x \geq 8
 \end{array}$$



The solution set is

$$\begin{aligned}
 &\{x \mid x < 5, \text{ or } x \geq 8\} \\
 &= (-\infty, 5) \cup [8, \infty) \\
 &= (-\infty, 5) \text{ OR } [8, \infty)
 \end{aligned}$$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Example 10: Solve: $3x - 5 \leq 13$ OR $5x + 2 > -3$

$$3x - 5 \leq 13$$

$$3x - 5 + 5 \leq 13 + 5$$

$$3x \leq 18$$

$$\frac{3x}{3} \leq \frac{18}{3}$$

$$x \leq 6$$

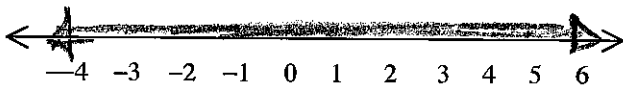
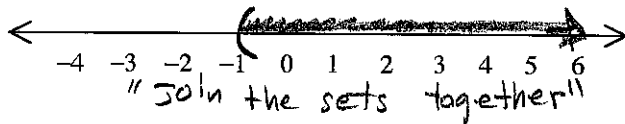
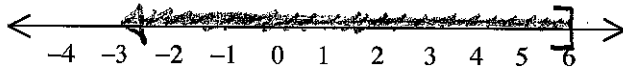
$$5x + 2 > -3$$

$$5x + 2 - 2 > -3 + (-2)$$

$$5x > -5$$

$$\frac{5x}{5} > \frac{-5}{5}$$

$$x > -1$$



The solution set is
 $\{x \mid -\infty < x < \infty\}$
 $= (-\infty, \infty)$.

Example 11: Solve: $2x - 7 > 3$ AND $5x - 4 < 6$

$$2x - 7 > 3$$

$$2x - 7 + 7 > 3 + 7$$

$$2x > 10$$

$$\frac{2x}{2} > \frac{10}{2}$$

$$x > 5$$

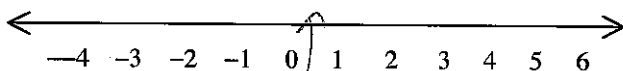
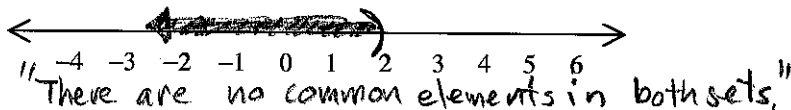
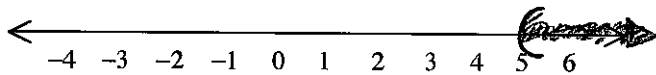
$$5x - 4 < 6$$

$$5x - 4 + 4 < 6 + 4$$

$$5x < 10$$

$$\frac{5x}{5} < \frac{10}{5}$$

$$x < 2$$



The graph has no points.

The solution set is
 $\{ \} = \emptyset$.

$$(-\infty, 2) \cap (5, \infty) = \emptyset$$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Answers Section 9.2

Example 1: $\{2, 4\}$

Example 2: \emptyset or $\{ \}$

Example 3: $(-3, 5)$

Example 4: $(-\infty, -4]$

Example 5: $(-4, 3]$

Example 6: $[-3, 1)$

Example 7: $\{1, 2, 3, 4, 5, 6\}$

Example 8: $\{1, 3, 4, 5, 6, 7, 8\}$

Example 9: $(-\infty, 5) \cup [8, \infty)$

Example 10: $(-\infty, \infty)$

Example 11: \emptyset or $\{ \}$

Section 9.3 Equations and Inequalities Involving Absolute Value

If an equation involves an unknown quantity that is enclosed in absolute value bars, to solve the equation you must rewrite the equation without the absolute value bars and then solve using an appropriate technique.

Rewriting an Absolute Value Equation without Absolute Value Bars

If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ or $X = -c$.

$$-X = c \text{ or } +X = c$$

Example 1: Solve the given absolute value equations.

a. $|3x - 7| = 1$

$3x - 7 = 1$ or $3x - 7 = -1$

My Preference: "Left"

$-(3x - 7) = 1$, or $+(3x - 7) = 1$

$-3x + 7 = 1$

$-3x + 7 - 7 = 1 - 7$

"left"

b. $|2x + 4| = 10$

$-(2x + 4) = 10$, or $+(2x + 4) = 10$

$-2x - 4 = 10$

$-2x - 4 + 4 = 10 + 4$

$-2x = 14$

$\frac{-2x}{-2} = \frac{14}{-2}$
 $x = -7$

$2x + 4 = 10$

$2x + 4 - 4 = 10 - 4$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

$-3x = -6$, or $3x = 8$

$\frac{-3x}{-3} = \frac{-6}{-3}$

$x = 2$

$\frac{3x}{3} = \frac{8}{3}$

$x = \frac{8}{3}$

Solution Set: $\{2, \frac{8}{3}\}$

Solution Set: $\{-7, 3\}$

c. $|4x - 5| + 3 = 17$ (Hint: First subtract 3 from both sides to isolate the absolute value term, and then rewrite the equation without absolute value bars using the rule in the box above Example 1.)

$-3 + |4x - 5| + 3 = -3 + 17$

$|4x - 5| = 14$

"left"

$-(4x - 5) = 14$

$-4x + 5 = 14$

$-4x + 5 - 5 = 14 - 5$

$-4x = 9$

$\frac{-4x}{-4} = \frac{9}{-4}$

$x = -\frac{9}{4}$

OR

"Right"

$+(4x - 5) = 14$

$4x - 5 = 14$

$4x - 5 + 5 = 14 + 5$

$4x = 19$

$\frac{4x}{4} = \frac{19}{4}$

$x = \frac{19}{4}$

check!

$x = -\frac{9}{4}$

$|4(-\frac{9}{4}) - 5| + 3 =$

$|-9 - 5| + 3 = 17$

$|-14| + 3 = 17$

$14 + 3 = 17$

$17 = 17$

TRUE!

check! $x = \frac{19}{4}$

$|4(\frac{19}{4}) - 5| + 3 = 17$

$|19 - 5| + 3 = 17$

$|14| + 3 = 17$

$14 + 3 = 17$

$17 = 17$

TRUE!

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Solution Set: $\{-\frac{9}{4}, \frac{19}{4}\}$

Solve Inequalities Involving Absolute Value

To solve an inequality involving absolute value, if the variable appears inside the absolute value symbols:

- Isolate the term involving the absolute value symbols.
- Rewrite the inequality without absolute values using the following rules:

$$|u| < a \text{ is equivalent to } -a < u < a \text{ (Also true for "≤")}$$

$$|u| > a \text{ is equivalent to } u < -a \text{ or } u > a. \text{ (Also true for "≥")}$$

- Solve the resulting inequalities.

Example 2: Solve the given inequalities. Express your solution in interval notation. Graph your solution on the number line.

a. $|x+4|+3 < 5$

$$-3 + |x+4| + 3 < -3 + 5$$

$$|x+4| < 2$$

My Preference!

"left"

$$-(x+4) < 2$$

$$-x - 4 < 2$$

$$-x - 4 + 4 < 2 + 4$$

$$-x < 6$$

$$-x \cdot (-1) > 6 \cdot (-1)$$

$$x > -6$$

AND

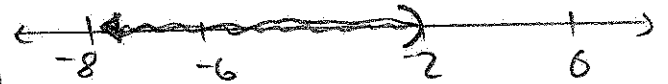
"Right"

$$+(x+4) < 2$$

$$x+4 < 2$$

$$x+4-4 < 2-4$$

$$x < -2$$



Solution Set:



$$\{x \mid -6 < x < -2\} = (-6, -2)$$

b. $|2-3x|-1 > 0$

$$|2-3x|-1+1 > 0+1$$

$$|2-3x| > 1$$

My Preference!

"left"

$$-(2-3x) > 1$$

$$-2+3x > 1$$

$$-2+3x+2 > 1+2$$

$$3x > 3$$

$$\frac{3x}{3} > \frac{3}{3}$$

$$x > 1$$

"Right"

$$+(2-3x) > 1$$

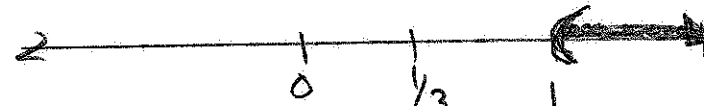
$$2-3x > 1$$

$$-2+2-3x > -2+1$$

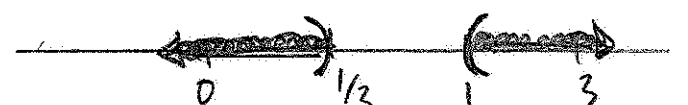
$$-3x > -1$$

$$\frac{-3x}{-3} < \frac{-1}{-3}$$

$$x < \frac{1}{3}$$



Solution Set:



$$\{x \mid x < \frac{1}{3} \text{ or } x > 1\} = (-\infty, \frac{1}{3}) \cup (1, \infty)$$

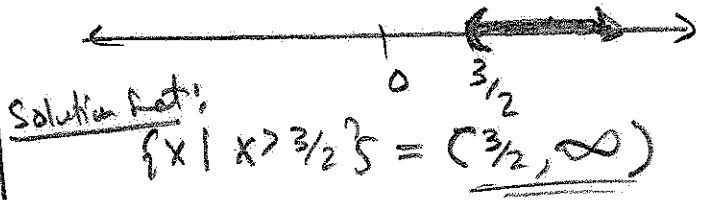
Extra Practice:

Solve the given equations or inequalities. Express your answer in interval notation.

Example 3:

a. $1 + 3x > 4 + x$

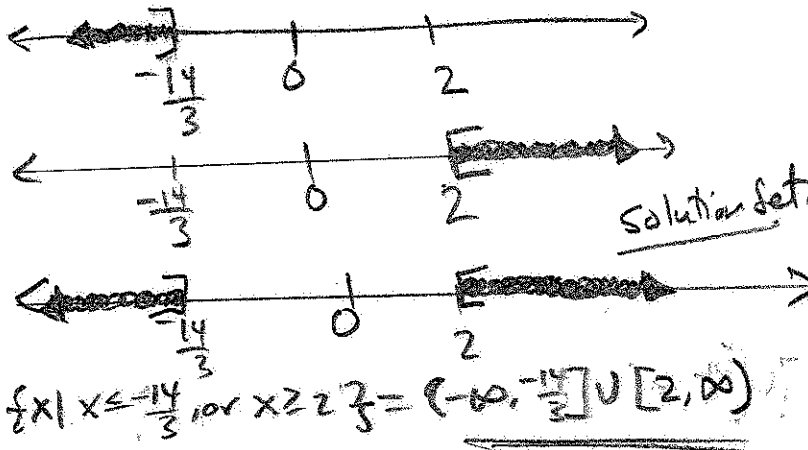
$$\begin{aligned} -x + 1 + 3x &> -x + 4 + x \\ 1 + 2x &> 4 \\ -1 + 1 + 2x &> -1 + 4 \\ 2x &> 3 \\ \frac{2x}{2} &> \frac{3}{2} \end{aligned} \Rightarrow \underline{x > \frac{3}{2}}$$



b. $|3x + 4| \geq 10$

"left" **OR** "Right"

$$\begin{aligned} -(3x + 4) &\geq 10 & + (3x + 4) &\geq 10 \\ -3x - 4 &\geq 10 & 3x + 4 &\geq 10 \\ -3x - 4 + 4 &\geq 10 + 4 & 3x + 4 - 4 &\geq 10 - 4 \\ -3x &\geq 14 & 3x &\geq 6 \\ \frac{-3x}{-3} &\leq \frac{14}{-3} & \frac{3x}{3} &\geq \frac{6}{3} \\ x &\leq -\frac{14}{3} & x &\geq 2 \end{aligned}$$



c. $|4 - 2x| = 18$

"left" **OR** "Right"

$$\begin{aligned} -(4 - 2x) &= 18 & + (4 - 2x) &= 18 \\ -4 + 2x &= 18 & 4 - 2x &= 18 \\ -4 + 2x + 4 &= 18 + 4 & -4 + 4 - 2x &= -4 + 18 \\ 2x &= 22 & -2x &= 14 \\ \frac{2x}{2} &= \frac{22}{2} & \frac{-2x}{-2} &= \frac{14}{-2} \\ x &= 11 & x &= -7 \end{aligned}$$

check: $x = 11$, $x = -7$

$$\begin{aligned} |4 - 2(11)| &= 18 & |4 - 2(-7)| &= 18 \\ |4 - 22| &= 18 & |4 + 14| &= 18 \\ |-18| &= 18 & |18| &= 18 \\ 18 &= 18 & 18 &= 18 \\ \text{TRUE!} & & \text{TRUE!} & \end{aligned}$$

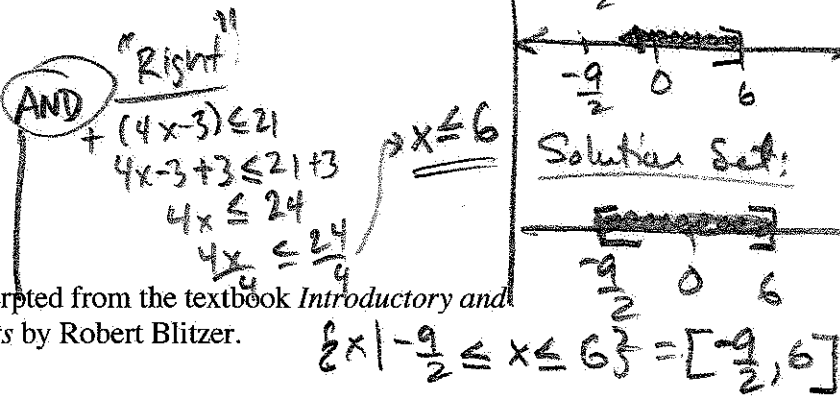
Solution set: $\{11, -7\}$

d. $|4x - 3| + 2 \leq 23$

$$\begin{aligned} -2 + |4x - 3| + 2 &\leq -2 + 23 \\ |4x - 3| &\leq 21 \end{aligned}$$

"left" **AND** "Right"

$$\begin{aligned} -(4x - 3) &\leq 21 & + (4x - 3) &\leq 21 \\ -4x + 3 &\leq 21 & 4x - 3 &\leq 21 + 3 \\ -4x + 3 - 3 &\leq 21 - 3 & 4x &\leq 24 \\ -4x &\leq 18 & \frac{4x}{4} &\leq \frac{24}{4} \\ \frac{-4x}{-4} &\geq \frac{18}{-4} & x &\leq 6 \end{aligned}$$



Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

$\text{1a; } \underline{\text{check; } x=2}$ $ 3(2)-7 =1$ $ 6-7 =1$ $ -1 =1$ $1=1$ <p style="text-align: center;">TRUE!</p>	$x = \frac{8}{3}$ $ 3(\frac{8}{3})-7 =1$ $ 8-7 =1$ $ 1 =1$ $1=1$ <p style="text-align: center;">TRUE!</p>	$\text{1b; } \underline{\text{check; } x=-7, x=3}$ $ 2(-7)+4 =10$ $ -14+4 =10$ $ -10 =10$ $10=10$ <p style="text-align: center;">TRUE!</p>	$ 2(3)+4 =10$ $ 6+4 =10$ $ 10 =10$ $10=10$ <p style="text-align: center;">TRUE!</p>
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Answers Section 9.3

Example 1a: $\{\frac{8}{3}, 2\}$

Example 1b: $\{3, -7\}$

Example 1c: $\{\frac{19}{4}, -\frac{9}{4}\}$

Example 2a: $(-6, -2)$ OR $\{x | -6 < x < -2\}$

Example 2b: $(-\infty, \frac{1}{3})$ or $(1, \infty)$

OR

$$\{x | x < \frac{1}{3} \text{ or } x > 1\}$$

Extra Practice:

Example 3a: $(\frac{3}{2}, \infty)$

Example 3b: $(-\infty, -\frac{14}{3}] \cup [2, \infty)$

Example 3c: $\{-7, 11\}$

Example 3d: $[-\frac{9}{2}, 6]$